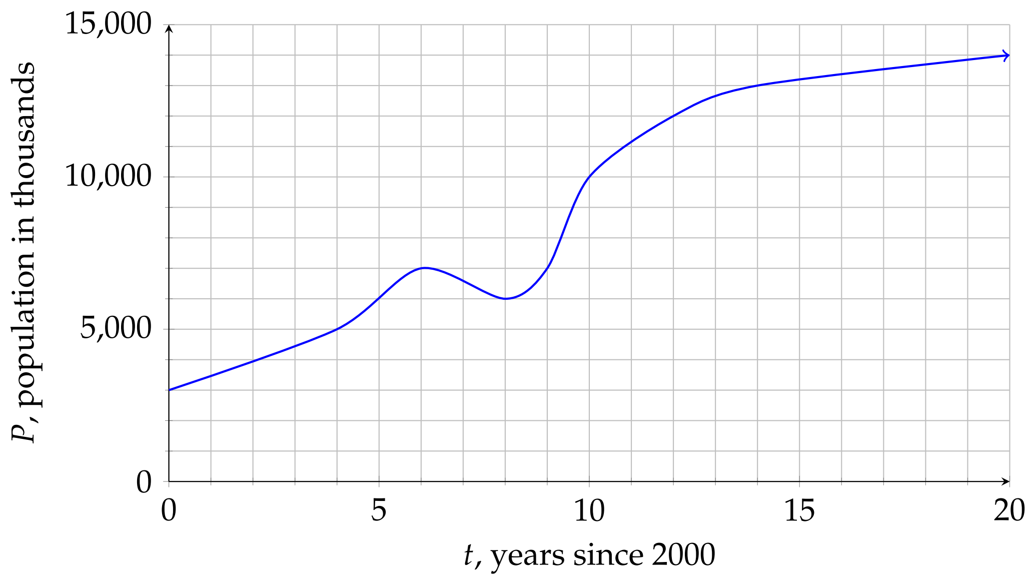
Math 117 - Fall 2021 - Common Final Exam, version A Solutions

1. The graph below shows P, the population of a region in thousands of people, as a function of t, years since 2000.



(a) (3 points) Estimate the average rate of change $\frac{\Delta P}{\Delta t}$ over the interval $0 \leq t \leq 10$.

Solution: $\frac{\Delta P}{\Delta t} \approx \frac{10{,}000-3{,}000}{10-0} = 700$

1 pt Evidence of using $\Delta f / \Delta t$

1 pt At least some progress toward correct solution

1 pt Correct solution

(b) (2 points) Give an interval of t values over which the average rate of change $\frac{\Delta P}{\Delta t}$ is negative.

(b) $6 < t < 8$

Solution: Any interval with a negative average rate of change is OK. 2 pts for a correct answer, 0 otherwise

(c) (2 points) Circle the interval below over which $\frac{\Delta P}{\Delta t}$ is **greatest**.

$0 \leq t \leq 10$ $\quad$ $6 \leq t \leq 10$ $\quad$ $8 \leq t \leq 10$

Solution: Correct choice: $8 \leq t \leq 10$. 2 points for correct choice, 0 otherwise.

(d) (2 points) Circle any intervals below over which the graph of P is concave up.

$2 \leq t \leq 4$ $\quad$ $7 \leq t \leq 9$ $\quad$ $12 \leq t \leq 14$

Solution: Correct choice: $7 \leq t \leq 9$. 2 points for correct choice, 0 otherwise.

2. A metal casting is being heat-treated in an industrial oven. The table below gives values for $h(t)$, the temperature of the casting t minutes after it is placed in the oven.

t (minutes)	0	8	16	24
$h(t)$ (° F)	1200	800	600	500

(a) (3 points) Find the average rate of change $\frac{\Delta h}{\Delta t}$ over the interval $0 \leq t \leq 8$. Give correct units and explain the meaning of this quantity in a sentence.

Solution: $\frac{\Delta h}{\Delta t} = \frac{800-1200}{8-0} = -50$ degrees F per minute. Over this interval, the casting is cooling at an average rate of 50 degrees per minute.

1 pt Correct average rate of change

1 pt Correct units

1 pt Substantially correct interpretation, at least referencing the interval somehow, or that this is an *average* rate of change.

(b) (3 points) Does h appear to be an increasing or decreasing function? Explain your answer in a sentence.

Solution: h appears to be decreasing, because the values of h decrease as t increases, or because the average rates of change are all negative.

1 pt Correctly identify function as decreasing

2 pt Correct explanation referencing at least one piece of evidence.

(c) (3 points) Does h appear to be a concave up or concave down function? Explain your answer in a sentence with some work to back it up.

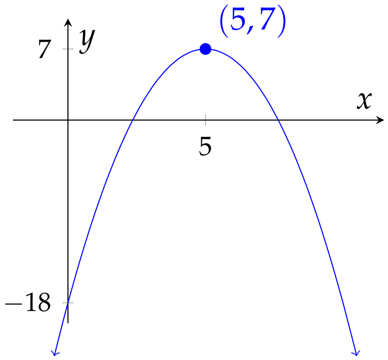
Solution: h appears to be concave up, because the average rate of change over successive intervals is increasing.

1 pt Correctly identify concavity

1 pt Answer is consistent with statement about increasing or decreasing average rate of change.

1 pt Some correct work is shown to back up statement about increasing or decreasing average rate of change.

3. (6 points) Write an expression for the quadratic function whose graph is given below.



Solution: $y = -(x-5)^2 + 7$

2 pt for some quadratic expression

2 pt correct coordinates for vertex

2 pt correct leading coefficient

4. Suppose that $C = g(t)$ is the concentration (in parts per billion or ppb) of nitrogen dioxide in a residential kitchen t minutes after igniting a natural gas cooktop.

   (a) (2 points) If you were to graph C as a function of t, which variable would appear on the horizontal axis?

   (a) ___ t ___

   Solution: 2 points for correct answer, 0 otherwise

   (b) (3 points) In a sentence, express the practical meaning of $g(15) = 200$.

   Solution: This means that 15 minutes after igniting the cooktop, the nitrogen dioxide concentration is 200 ppb.

   1pt Evidence of identifying the input and output of the function, but may not put these values in the correct practical context.

   1 pt Evidence of correct context, but may have some errer, e.g. incorrect units.

   1 pt All's well.

   (c) (2 points) What are the units of the average rate of change $\frac{\Delta C}{\Delta t}$?

   Solution: Units of $\frac{\Delta C}{\Delta t}$ are ppb per minute. 2 points for correct units, 0 points otherwise.

5. (5 points) Find the slope and intercept of the line represented by $2x + 4y = 3(x-8)$, and write them in the spaces provided below.

Solution: $y = \frac{1}{4}x - 6$

1 pt Evidence of manipulating the equation to get slope and intercept terms.

2 pts Progress shown toward correct algebra.

1 pt Correct slope identified.

1 pt Correct intercept identified.

Note: correct slope and intercept values alone should be considered evidence of sufficient work; it is possible that this could be solved without pencil-and-paper work.

(a) The slope is $\underline{\quad \frac{1}{4} \quad}$.

(b) The intercept is $\underline{\quad \textbf{-6} \quad}$.

6. The cost, in dollars, for producing x cuckoo clocks is $C(x) = 1200 + 45x$.

(a) (3 points) Find a formula for the inverse $C^{-1}(y)$.

Solution: $C^{-1}(y) = \frac{y-1200}{45}$

1 pt Progress toward algebraic solution for inverse.

1 pt Correct expression for C^{-1} but there may be mismatched variables.

1 pt Consistent variable for expression for C^{-1}.

(b) (3 points) Explain in a sentence the meaning of the quantity $C^{-1}(6150)$.

Solution: $C^{-1}(6150)$ is the number of cuckoo clocks produced for a cost of $6150.

1 pt Answer includes some reference to 6150 as input to the inverse function.

1 pt Answer explains that 6150 is a cost in dollars.

1 pt Answer correctly identifies inverse function value as a number of cuckoo clocks.

7. In 2000, the North Bay fishing industry brought in 550 tons of fish. The annual catch in North Bay is decreasing linearly by 20 tons per year.

(a) (3 points) Write an expression for $g(t)$, the quantity of fish caught by the North Bay fishing industry (in tons) t years after 2000.

Solution: $g(t) = 550 - 20t$

1 pt Some linear expression is given.

1 pt Correct slope

1 pt correct intercept

(b) (4 points) How long will it take for the quantity to decrease to 0? Show work to support your answer.

Solution: $t =$ 27.5 years

1 pt Evidence of setting $g(t) = 0$

1 pt progress toward solution

1 pt correct answer

8. The graph of $y = f(x)$ passes through the points $(2, 4)$ and $(6, 108)$.

(a) (4 points) Find an expression for $f(x)$ if f is a linear function of the form $f(x) = mx + b$.

Solution: $y = 26x - 48$

1 pt Answer gives some linear expression

1 pt Progress toward solution

1 pt Slope is correct

1 pt Intercept is correct

(b) (4 points) Find an expression for $f(x)$ if f is a power function of the form $f(x) = kx^p$.

Solution: $y = \frac{1}{2} \cdot x^3$

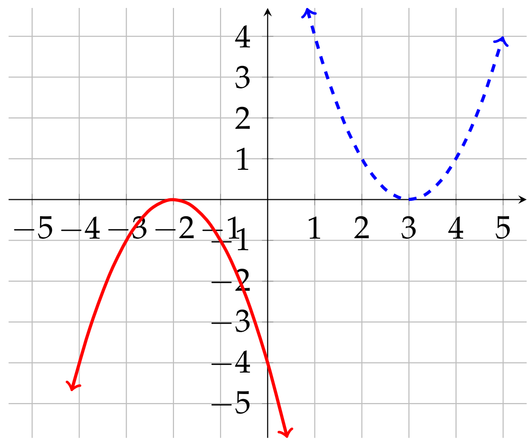
1 pt Answer gives some power function

1 pt Progress toward solution

1 pt Power is correct

1 pt Coefficient is correct

9. Below is the graph of $y = f(x)$ (solid line) and the graph of $y = g(x)$ which is obtained by some transformations of f.



(a) (4 points) Describe the transformations in words.

Solution: Shift to the right by 5 units and reflect about the x axis.

2 pt Correct horizontal transformation

2 pt Correct vertical transformation

(b) (3 points) Write a formula for $g(x)$ in terms of $f(x)$.

Solution: $g(x) = -f(x-5)$

1 pt Transformation is at least horizontal/"inside" , possibly incorrect sign

1 pt Correct vertical transformation

1 pt Sign for horizontal shift is OK.

10. Ellen was hired at the start of 2018. Her contract states that her hourly pay will be $f(t)$ dollars t months after she was hired.

(a) (3 points) Louie is hired 6 months after Ellen, and is paid according to the same pay scale as Ellen. Write an expression in terms of f for Louie's pay t months after the start of 2018.

Solution: $f(t-6)$

1 pt Some horizontal transformation ("inside")

1 pt Some shift, not stretch

1 pt sign is OK

(b) (3 points) Sara was hired at the same time as Ellen, and the pay for her position is twice as much as Ellen's position for the same amount of seniority. Write an expression in terms of f for Sara's pay t months after the start of 2018.

Solution: $2f(t)$

1 pt Some vertical transformation (“outside”)

1 pt Some stretch, not shift

1 pt $\times 2$, not $\times \frac{1}{2}$

(c) (4 points) Suppose $f(8) = 12$. Fill in the blanks with values you can infer from this.

- **8** months after the start of 2018, Ellen’s pay is **12** dollars per hour.
- **8** months after the start of 2018, Sara’s pay is **24** dollars per hour.

Solution: Blanks as filled in; 1 point each.

11. (4 points) The point $(-3, 8)$ is on the graph of $y = f(x)$. Give the coordinates of one point on the graph of $y = -f(\frac{1}{2}(x+1))$.

Solution: $(-7, -8)$

2 pts Correct x coordinate

2 pts Correct y coordinate

12. A researcher has discovered a revolutionary new material with the property that its electrical resistance (measured in Ohms) is proportional to the square root of its temperature (measured in degrees Celsius).

(a) (4 points) Find an expression for the material’s resistance R as a function of its temperature T. Your answer will contain a constant k.

Solution: $R = k\sqrt{T}$

1 pt There is some proportional relationship, maybe inverse, for R and T

1 pt $\sqrt{T}$ is used

2 pt Correct answer

(b) (3 points) The researcher determines that when the material’s temperature is 64°C, its resistance is 280 Ohms. Find k and rewrite your answer from part a) using it.

Solution: Solve $280 = k\sqrt{64}$ for k to get $R = 35\sqrt{T}$.

1 pt Set up an equation to solve for k

1 pt equation for k is correct given expression from the previous part

1 pt Algebra is OK and answer is given as a relation between R and T.

(c) (3 points) At what temperature does the material have a resistance of 105 Ohms?

Solution: Solve $105 = 35\sqrt{T}$ to get $T = 9$.

1 pt Set up equation to solve for T

1 pt Progress toward solution

1 pt All's well

13. Consider the polynomial function $y = (x^2 - 16)(x^2 + 2x - 3)$.

(a) (2 points) Find the leading term.

Solution: x^4; 1 pt for at least correct degree or correct expansion without indicating leading term; 1 point for correctly indicating the leading term

(b) (2 points) Find the degree.

Solution: 4; 1 point for at least giving leading term with correct degree; 1 point for only the degree

(c) (4 points) Find all zeros.

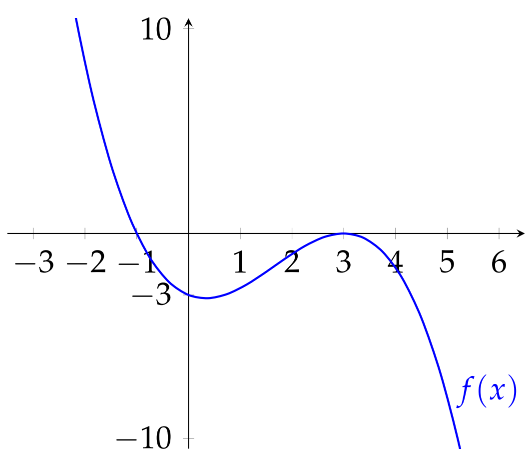
Solution: Finish factoring $y = (x-4)(x+4)(x+3)(x-1)$. Zeros at $x = -4, -3, 1, 4$.

1 pt Evidence of correct factoring (it is OK if difference of squares is not factored; correct zeros could be evidence of correct factoring)

1 pt All zeros present possibly with sign errors

1 pt All zeros present and signs are correct

14. (4 points) Find an expression of minimum degree for the polynomial graphed below.



Solution: $y = -\frac{1}{3}(x+1)(x-3)^2$

1 pt Answer is a polynomials with at least some zeros are accounted for as factors

1 pt Double zero appears correctly

1 pt Degree is correct

1 pt Correct leading coefficient

15. Suppose that the rabbit population on Mr. Jenkins' farm follow the formula

$$p(t) = \frac{3000t + 200}{t+1}$$

where $t \geq 0$ is the time (in years) since he started farming.

(a) (3 points) What is the population of rabbits when Mr. Jenkins starts farming?

Solution: $p(0) = 200$

1 pt Evidence of evaluating $p(0)$, but there may be some error

1 pt progress toward evaluation

1 pt All's well

(b) (4 points) Fill in the following limit statement, and then write a sentence to explain what happens to the rabbit population in the long run.

$$\lim_{t\to\infty} \frac{3000t + 200}{t+1} = \underline{\quad \textbf{3000} \quad}$$

Solution: In the long run, the rabbit population approaches 3000 rabbits.

2 pt Correct limit value

2 pt Correct expanation referencing "population approaches" or "limiting population" or something like that.

(c) (3 points) How long does it take for the rabbit population to reach 1000?

Solution: Solve $p(t) = 1000$ to get $t = \frac{2}{5}$ year.

1 pt Evidence of setting up equation $p(t) = 1000$, but solution may be incorrect

1 pt Progress on solution, but there may be some error

1 pt All's well

16. Let $f(x) = \dfrac{(x-1)(x-2)(x+5)}{(x+7)(2x+9)(x+5)}$.

(a) (2 points) Find the x coordinates of the holes (if any) in the graph of $y = f(x)$.

Solution: $x = -5$, 1 point for correct value, 1 point for correct sign

(b) (2 points) Find the equation for each vertical asymptote, if any.

Solution: $x = -7$ and $x = -\frac{9}{2}$; 1 point for something correct, 2 points for both asymptotes with correct notation

(c) (2 points) Find the equation for each horizontal asymptote, if any.

Solution: y = 0.5; 1 point for correct value; 1 point for correct expression as a line

(d) (2 points) Find the x-intercepts, if any.

Solution: 1 and 2, 1 point each

(e) (2 points) Find the y-intercept, if any.

Solution: $\frac{2}{63}$; 1 point for evicdence of evaluating $f(0)$; 1 point for correct work